Performance-based Topology Optimization

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摘要

傳統結構拓樸最佳化方法，乃於特定結構體積下尋找能滿足結構最小順度(compliance)之最佳拓樸形狀，而不考慮其他功能性的限制條件，諸如：位移、應力、頻率等。然而在實務設計上，結構系統通常必須考慮各種不同功能性的限制條件，導致傳統拓樸最佳化概念在應用上受到頗多限制。本研究提出效能導向(Performance-based)拓樸最佳化問題，此類問題的目標為尋求最小化結構順度，同時滿足預設的功能性限制條件之最佳化拓樸形狀。為此本文發展出效能導向之元素演進交換法PESM(performance-based evolutionary switching method)，藉由演進法調整結構體積，再透過交換法調整拓樸形狀，使之達到在滿足功能性限制式下，最小化結構順度之最佳拓樸形狀。最後，本研究應用有限元素軟體Abaqus實作PESM，以數值實例探討不同之功能性限制條件對最佳化拓樸形狀之影響。

關鍵字：拓樸最佳化、效能導向、功能性限制條件、元素演進交換法、有限元素

ABSTRACT

Conventional topology optimization can find out a structure’s optimal topology that satisfies both the minimum compliance and a specified volume constraint. Nevertheless, such concept of topology optimization ignores practical design requirements, which often include different functional constraints. Confronting that ignorance, this paper proposes problems of performance-based topology optimization, problems looking for an optimal topology that satisfies both the minimum compliance and pre-specified functional constraints. In addition, this paper creates a novel element-exchange-based method, which is able to solve these performance-based problems by adjusting a structure’s volume automatically. The method is called the performance-based evolutionary switching method (PESM) and implemented with a famous commercial finite element software, Abaqus. Finally, numerical examples are represented and solved to investigate how different functional constraints change the optimal topology of a structure.

Keywords: topology optimization, performance-based, functional constraint, element exchange method, finite element

1. INTRODUCTION

Structural optimization is often divided into three types: sizing, geometrical, and topology optimization (Rozvancy 1993). For sizing and geometrical optimization the topology of a structure cannot be changed during the optimization process. It is not guaranteed that the structure obtained is the best or even a good one since another initial topology might produce a remarkably better solution for the optimization problem. However, topology optimization looks for the best topology satisfying the volume constraint from an initial design domain. Therefore, the final optimization solution will be, indeed, the best structure compared with the solutions obtained from the other two types of optimization methods (Tanskanen 2002).

The conventional continuum-based topology optimization can be interpreted to seek an optimal distribution of a fixed amount of material over a larger reference domain through specified objective and constraint functions. In general, the corresponding objective function is the minimization of the compliance for a linearly elastic structure. The compliance of a structure is often defined as twice the work done by the external force and can be expressed in the following matrix form:

$$\text{Min. } C_p = \mathbf{P}^T\mathbf{U} \tag{1}$$

where P and U represent the global force vector and displacement vector. C_p denotes the compliance of the structure. The displacement vector in general is an implicit function of design variables and is obtained by solving the equilibrium equation, Eq. (2), in the finite element method.

$$\mathbf{KU} = \mathbf{P} \tag{2}$$

where K is the stiffness matrix of the structure. Moreover, the volume of material must satisfy the prescribed volume limit $(V)_{specified}$.

$$V \leq (V)_{specified} \tag{3}$$

However, there still existed some deficiencies in the mathematical model of conventional topology optimization problems. First, there are no functional constraints, such as displacements, stresses, or natural frequencies. In other words, that satisfies no engineering requirements in practical applications. Second, from the viewpoint of a designer, the “optimal” value of the volume in general is unknown in priori. The prescribed volume limits are often criticized and seems to violate the common sense in practical design procedures.

In order to overcome the limitations of the conventional topology optimization, this paper proposes problems of performance-based topology optimization. In any of such problems, the objective function is defined as the minimization of the compliance for a linearly elastic structure. The volume constraint, however, is removed. The functional constraints are added. The corresponding mathematical model can be written as:

$$\text{Min. } C_p = \mathbf{P}^T\mathbf{U} \tag{4a}$$

$$\text{Sb. } f_j(\mathbf{u}) \leq (f_j)_{allow} \tag{4b}$$

where **u** is the nodal displacement vector and $(f_j)_{allow}$ is the allowable value for the $j^{\text{-th}}$ functional constraint. In brief, a performance-based topology optimization problem focuses on required engineering performance (e.g., allowable displacement or stress) while the final topology is still with the minimum compliance. To solve such a problem, this paper creates the performance-based evolutionary switching method (PESM).

2. METHOD

To consider functional constraints during topology optimization, the optimization method should be able to adjust the volume of material in the design domain so that the constraints can be satisfied. PESM uses double loops to make it. The outer loop is used to adjust the volume of material to satisfy the functional constraints. The inner loop is used to find the optimal topology with the minimum compliance under a specified volume of material which is determined from the outer loop. Furthermore, the inner loop can be divided into two stages: the evolutionary stage and the switching stage. In the evolutionary stage, the volume of material in the initial topology can be different from the volume of material in the final topology. The goal of this stage is to adjust the volume of material in the initial topology to the pre-assumed volume of material. In the switching stage, the distribution of material is adjusted to achieve the optimal topology with minimum compliance under the pre-assumed volume of material. After the end of the inner loop, the constraint requirements are then checked to determine how to adjust the volume of material in the outer loop. The procedure of the proposed PESM is depicted in Fig. 1 and more details are discussed in the following.

3. NUMERICAL EXAMPLES

In the following examples, the proposed PESM is used to minimize the compliance of the structure with a vertical displacement constraint. The common conditions of the examples are the elastic modulus E_{solid} = 200 GPa, and Poisson’s ratio = 0.3. To avoid ill-conditioning of the stiffness matrix, we assume all void elements have very small elastic moduli, i.e. $E_{\text{void}} = 10^{-4}\ E_{\text{solid}}$. In the first illustrative example, a two-dimensional beam is considered. This example is concerned with the two-dimensional Michell type structure, which has been solved analytically as a benchmark problem for elastic topology optimization. As shown in Fig. 2, a design domain with the dimension of 0.3×0.1 m^2 is discretized into 1200 quadratic plane stress elements (CPS8 in Abaqus), each of which have size of 0.005×0.005 m^2. The two corners at the bottom are assumed rollers. A concentrated load, P = 10000 N, is applied at the point A, the middle of the bottom. Also at the point A, the only one constraint in the example is given as the point's allowable vertical displacement. Linear elastic static analyses are performed to calculate the corresponding element strain energy density.

Fig. 3 shows an optimal topology solved by PESM when the point A's allowable displacement is 2.5×10^{-4} m. Fig. 4 shows another optimal topology solved by PESM when the allowable displacement is 6×10^{-4} m. The two topologies are similar to those by ESO (Xie and Steven 1993). The fact implies PESM could solve conventional topology problems. In addition, the volume ratios of material in the design domain are 48.33 % and 21.83 % corresponding to the smaller allowable displacement and the larger allowable displacements, respectively. At the point A, the solved displacements of the two optimal topologies are 2.493×10^{-4} m and 5.995×10^{-4} m. Both satisfy and approximate the two allowable displacements. This example demonstrates that by adjusting the material volume ratio, PESM is able to satisfy the displacement constraint and to get the optimal topology with the minimum compliance.

4. CONCLUSIONS

Compared with conventional topology optimization, performance-based topology optimization is closer to practical design applications. Engineers only need to specify the functional requirements while the material volume ratio is adjusted automatically to meet these requirements. In this study, the performance-based evolutionary switching method (PESM) is proposed to solve problems of performance-based topology optimization. PESM is a double-loop algorithm where the outer loop adjusts the volume of material and the inner loop finds the optimal topology with minimum compliance.

Numerical examples show that PESM could solve problems of performance-based topology optimization. For structures subjected to a single concentrated load with displacement constraint in the same direction of the applied load, the final topologies by PESM has the same trend as those by other methods, and the values of allowable displacements dominate the final volumes of materials.

REFERENCE

[1] Rozvancy, G.I.N. (1993), "Layout theory for grid-type structures," Topology Design of Structures, edited by Bendsøe, M.P. and Mota Soares, C.A., Kluwer Academic Publishers, London, 251-272.

[2] Tanskanen, P. (2002), "The evolutionary structural optimization method: theoretical aspects," Comput. Methods Appl. Mech. Eng., 191, 5485-5498.

[3] Xie, Y.M. and Steven, G.P. (1993), "A simple evolutionary procedure for structural optimization", Comput. Struct., 49(5), 885-896.

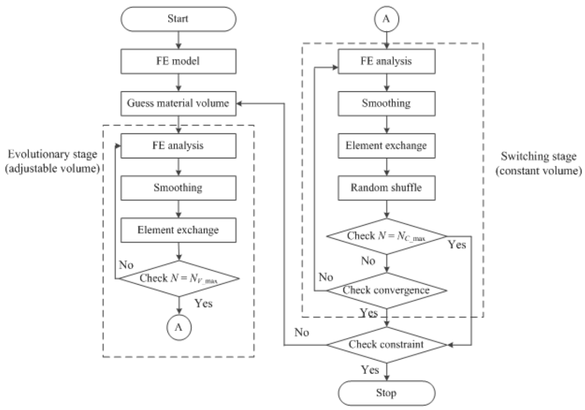

Fig. 1 Flowchart of PESM

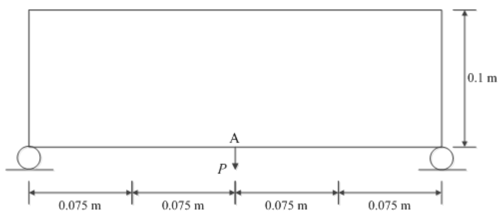

Fig. 2 Design domain of the Michell type structure with a concentrated load

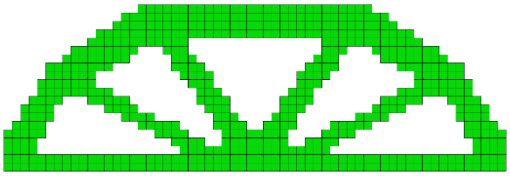

Fig. 3 Optimal topology for $(u_j)_{allow}$ = 2.5×10^{-4}m

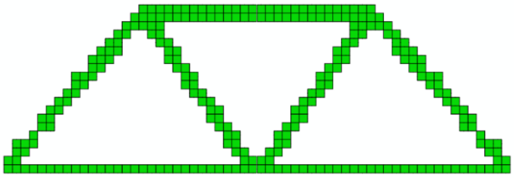

Fig. 4 Optimal topology for $(u_j)_{allow}$ = 6×10^{-4}m